It was shown above (1.44) that this time-dependent asymmetry is given by:

\[ a_{f_{CP}} = \frac{(1 \leftrightarrow |\lambda_{f_{CP}}|^2) \cos(\Delta m_B t) \leftrightarrow 2 \Im \lambda_{f_{CP}} \sin(\Delta m_B t)}{1 + |\lambda_{f_{CP}}|^2} \].

(1.59)

This asymmetry will be non-vanishing if any of the three types of \( CP \) violation are present. However, for decays such that \( |\lambda| = 1 \) (the ‘clean’ modes — see below), (1.44) simplifies considerably:

\[ a_{f_{CP}} = \leftrightarrow \Im \lambda_{f_{CP}} \sin(\Delta m_B t) \].

(1.60)

One point concerning this type of asymmetries is worth clarifying. Consider the decay amplitudes of \( B^0 \) into two different final \( CP \) eigenstates, \( A_a \) and \( A_b \). A non-vanishing difference between \( \eta_a \lambda_a \) and \( \eta_b \lambda_b \),

\[ \eta_a \lambda_a \leftrightarrow \eta_b \lambda_b = \frac{q}{p} \left( \frac{A_{1\pi}}{A_a} \leftrightarrow \frac{A_{1\pi}}{A_b} \right) \neq 0, \]

(1.61)

would establish the existence of \( CP \) violation in \( \Delta b = 1 \) processes. For this reason, this type of \( CP \) violation is also called sometimes “direct \( CP \) violation.” Yet, unlike the case of \( CP \) violation in decay, no nontrivial strong phases are necessary. The richness of possible final \( CP \) eigenstates in \( B \) decays makes it very likely that various asymmetries will exhibit (1.61). (A measurement of \( \mathcal{B}(K_L \to \pi \nu \bar{\nu}) \gtrsim 10^{-11} \) can establish the existence [17, 18, 19] of a similar effect, a \( \Delta s = 1 \) \( CP \) violation that does not depend on strong phase shifts.) Either this type of observation or the observation of \( CP \) violation in decay would rule out superweak models for \( CP \) violation.

\( CP \) violation in the interference between decays with and without mixing can be cleanly related to Lagrangian parameters when it occurs with no \( CP \) violation in decay. In particular, for \( B_d \) decays that are dominated by a single \( CP \)-violating phase, so that the effect of \( CP \) violation in decay is negligible, \( a_{f_{CP}} \) is cleanly translated into a value for \( \Im \lambda \) which, in turn, is cleanly interpreted in terms of purely electroweak Lagrangian parameters. (As discussed below, \( \Im \varepsilon_K \) which describes \( CP \) violation in the interference between decays with and without mixing in the \( K \) system, is cleanly translated into a value of \( \phi_{12} \), the phase between \( M_{12}(K) \) and \( \Gamma_{12}(K) \). It is difficult, however, to interpret \( \phi_{12} \) cleanly in terms of electroweak Lagrangian parameters.)

When there is \( CP \) violation in decay at the same time as in the interference between decays with and without mixing, the asymmetry (1.58) depends also on the ratio of the different amplitudes and their relative strong phases, and thus the prediction has hadronic uncertainties. In some cases, however, it is possible to remove any large hadronic uncertainties by measuring several isospin-related rates (see e.g., [20, 21, 22]) and thereby extract a clean measurement of CKM phases. This is discussed in further detail in Chapters 5 and particularly 6.

There are also many final states for \( B \) decay that have \( CP \) self-conjugate particle content but are not \( CP \) eigenstates because they contain admixtures of different angular momenta and hence different parities. In certain cases angular analyses of the final state can be used to determine the amplitudes for each different \( CP \) contribution separately. Such final states can then also be used for clean comparison with theoretical models [23]. This is discussed in more detail in Chapter 5.